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Electron-pair condensation in parity-preserving QED$_3$*

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Abstract

In this paper, we present a parity-preserving QED$_3$ with spontaneous breaking of a local $U(1)$-symmetry. The breaking is accomplished by a potential of the φ^6-type. It is shown that a net attractive interaction appears in the Møller scattering (s and p-wave scattering between two electrons) as mediated by the gauge field and a Higgs scalar. This might favour a pair-condensation mechanism.

1 Introduction

Over the past years, the study of 3-dimensional field theories [1] has been well-supported in view of the possibilities they open up for the setting of a gauge-field-theoretical foundation in the description of Condensed Matter phenomena, such as High-T_c Superconductivity [2] and Quantum Hall Effect [3]. Abelian models such as QED$_3$ and τ_3QED$_3$ [4, 5] are some of the theoretical approaches proposed to describe more deeply some features of high-T_c materials.

The theory of superconductivity by Bardeen, Cooper and Schrieffer (BCS model) [6] succeeds in providing a microscopical description for superconducting materials: indeed, many predictions of the BCS model have been confirmed experimentally. An elegant mathematical formulation was given to it by Bogoliubov [7]. The characteristic feature of the BCS theory is that it produces an energy gap between the ground state and the excited states of a superconductor. The gap is due to the fact that the attractive phonon-mediated interaction between electrons produces correlated pairs of such particles (Cooper pairs) [8], with opposite momenta and spin; a finite amount of energy is required to break this correlation.

In a well-known paper by Nambu and Jona-Lasinio [9], it was proposed that the nucleon mass might arise from a dynamical mechanism, similar to the appearance of the energy gap in the BCS model. They proposed that elementary excitations in a superconductor could be described by means of a coherent mixture of electrons and holes. The

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framework they set up for dynamical mass generation was motivated by the observation of an analogy between the properties of Dirac particles and the quasi-particle excitations that appear in a superconductor.

The main purpose of this paper is to show that electrons scattered in D=1+2 can experience a mutual net attractive interaction, not depending on their spin states. This attractive scattering potential comes from processes in which the electrons are correlated in momentum space with opposite spin polarisations (s-wave state). Also, in the case of equal spin polarisations (p-wave state), a net attraction may appear, as due to the Higgs interaction, if some special conditions are set up on the parameters. The latter possibility should be investigated for the cases in which very high external magnetic fields are applied, since it is suspected that the resistance of the superconducting state in the presence of high magnetic fields, in the re-entrant superconductivity effect, could be explained by p-wave states, p-electron pairing [10]. The intermediate bosons are a massive vector meson and a Higgs scalar, both resulting from the breaking of a local $U(1)$-symmetry. The breakingdown is accomplished by a sixth-power potential. We analyse the conditions on the parameters in order to avoid metastable vacuum states. The method used here to compute the scattering potentials is based on the ideas reported in a series of papers by Sucher *et al.* [11]. The behaviour of the scattering interactions mediated by the massive vector meson and the Higgs scalar are presented for electrons scattered in s and p-wave processes. The interesting feature of s and p scatterings, since net attractive potentials are generated, motivated the study of Bethe-Salpeter equation [12, 13] associated to the model proposed here [14], in order to verify, if whether or not there are s and p-wave bound states. The issue of confinement in QED$_3$ [15] is also alluded to. The behaviour at the quantum level of the model proposed in this letter, in the symmetric and broken regimes, is analised in ref.[16] by using the algebraic renormalisation method, which is independent of any kind of regularisation scheme [17].

The outline of our paper is as follows. In Section 2, the parity-preserving Abelian model is presented as the spontaneous breaking of the $U(1)$-symmetry is discussed in the R_ξ-gauge. Next, in Section 3, the calculation of the tree-level scattering amplitude is presented and the net attractive potential is worked out. Finally, in Section 4, we cast a few remarks on our results. One Appendix follows, where a few comments on Dirac fermions in D=1+2 are pointed out.

2 Parity-preserving QED$_3$ coupled to scalar matter

The action for the parity-preserving QED$_3$ [1] with spontaneous symmetry breaking of a local $U(1)$-symmetry is given by :

$$S_{\rm QED} = \int d^3x \Big\{ -\frac{1}{4}F^{mn}F_{mn} + i\overline{\psi}_+ \not{D}\psi_+ + i\overline{\psi}_- \not{D}\psi_- - y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)\varphi^*\varphi + \\ + D^m\varphi^* D_m\varphi - V(\varphi^*\varphi)\Big\} \qquad , \tag{1}$$

with the potential $V(\varphi^*\varphi)$ taken as

$$V(\varphi^*\varphi) = \mu^2\varphi^*\varphi + \frac{\zeta}{2}(\varphi^*\varphi)^2 + \frac{\lambda}{3}(\varphi^*\varphi)^3 \qquad , \tag{2}$$

[1] The metric adopted throughout this work is $\eta_{mn} = (+,-,-)$; m, n=(0,1,2). Note that slashed objects mean contraction with γ-matrices. The latter are taken as γ^m=$(\sigma_x, i\sigma_y, -i\sigma_z)$.

where the mass dimensions of the parameters μ, ζ, λ and y are respectively 1, 1, 0 and 0.

The covariant derivatives are defined as follows :

$$\not{D}\psi_\pm \equiv (\not{\partial} + iqg\not{A})\psi_\pm \quad \text{and} \quad D_m\varphi \equiv (\partial_m + iQgA_m)\varphi \quad , \tag{3}$$

where g is a coupling constant with dimension of $(\text{mass})^{\frac{1}{2}}$ and, q and Q are the $U(1)$-charges of the fermions and scalar, respectively. In the action (1), F_{mn} is the usual field strength for A_m, ψ_+ and ψ_- are two kinds of fermions (the $\pm$ subscripts refer to their spin sign [18], see also the Appendix) and φ is a complex scalar. The $U(1)$-symmetry gauged by A_m is interpreted as the electromagnetic one, so that A_m is meant to describe the photon. It is noteworthy to remark that terms of the form $\psi_\pm^\alpha\psi_{\pm\alpha}\varphi\varphi$ and $\psi_\pm^\alpha\psi_{\pm\alpha}\varphi^*\varphi^*$ are not adjoined to the interaction Lagrangian because Lorentz invariance would require the fermion to be Majorana [2]. However, if such were the case these terms would explicitly break the $U(1)$-invariance, unless there would be more than a flavour of scalars. So, since we are dealing with Dirac fermions and just a complex scalar, the term $\overline{\psi}_\pm\psi_\pm\varphi^*\varphi$ is indeed the only one that couples fermions to scalars in a way compatible with Lorentz and gauge invariance while respecting renormalisability.

The QED$_3$-action[3] (1) is invariant under the discrete symmetry, P, whose action is fixed below :

$$x_m \xrightarrow{P} x_m^P = (x_0, -x_1, x_2) \quad , \tag{4.a}$$

$$\psi_\pm \xrightarrow{P} \psi_\pm^P = -i\gamma^1\psi_\mp \quad , \quad \overline{\psi}_\pm \xrightarrow{P} \overline{\psi}_\pm^P = i\overline{\psi}_\mp\gamma^1 \quad , \tag{4.b}$$

$$A_m \xrightarrow{P} A_m^P = (A_0, -A_1, A_2) \quad , \tag{4.c}$$

$$\varphi \xrightarrow{P} \varphi^P = \varphi \quad . \tag{4.d}$$

Since we are looking for a model that preserves the parity and time-reversal in D=1+2, it should be noticed that the transformation (4.c) has been imposed in such a way that the interactions respect both invariances.

The sixth-power potential, V, is the responsible for breaking the electromagnetic $U(1)$-symmetry. It is the most general renormalisable potential in $3D$.

Analysing the potential (2), and imposing that it is bounded from below and yields only stable vacua (metastability is ruled out), the following conditions on the parameters μ, ζ, λ must be set :

$$\lambda > 0 \quad , \quad \zeta < 0 \quad \text{and} \quad \mu^2 \leq \frac{3}{16}\frac{\zeta^2}{\lambda} \quad . \tag{5}$$

We denote $\langle\varphi\rangle$=v and the vacuum expectation value for the $\varphi^*\varphi$-product, v^2, is chosen as

$$\langle\varphi^*\varphi\rangle = v^2 = -\frac{\zeta}{2\lambda} + \left[\left(\frac{\zeta}{2\lambda}\right)^2 - \frac{\mu^2}{\lambda}\right]^{\frac{1}{2}} \quad , \tag{6}$$

the condition for minimum being read as

$$\mu^2 + \zeta v^2 + \lambda v^4 = 0 \quad . \tag{7}$$

[2]For Dirac fermions (ψ) one has $\overline{\psi}\equiv\overline{\psi}^\alpha$=$-C^{\alpha\beta}\psi_\beta^c$, since for Majorana fermions (θ) θ^c=θ, then it follows that $\overline{\theta}\equiv\overline{\theta}^\alpha$=$-C^{\alpha\beta}\theta_\beta$. Therefore, for Majorana fermions $\overline{\theta}\theta$=$\theta^\alpha\theta_\alpha$.

[3]For more details about QED$_3$ and τ_3QED$_3$, as well as their applications and some peculiarities of parity and time-reversal in D=1+2, see refs. [1, 4, 5].

The complex scalar, φ, is parametrised by

$$\varphi = v + H + i\theta \quad , \tag{8}$$

where θ is the would-be Goldstone boson and H is the Higgs scalar, both with vanishing vacuum expectation values. It should be noticed that the parametrisation given by eq.(8) was chosen in order to avoid non-renormalisable interactions [19].

By replacing the parametrisation (8) for the complex scalar, φ, into the action (1), the following free action comes out:

$$\begin{aligned}\hat{S}_{\mathrm{QED}}^{\mathrm{free}} = \int d^3x \Big\{ & -\frac{1}{4}F^{mn}F_{mn} + \frac{1}{2}M_A^2 A^m A_m + \overline{\psi}_+(i\partial\!\!\!/ - m)\psi_+ + \overline{\psi}_-(i\partial\!\!\!/ + m)\psi_- + \\ & + \partial^m H \partial_m H - M_H^2 H^2 + \partial^m\theta\partial_m\theta + 2vQgA^m\partial_m\theta \Big\} \quad , \end{aligned} \tag{9}$$

where the parameters M_A^2, m and M_H^2 are given by

$$M_A^2 = 2v^2Q^2g^2 \quad , \quad m = yv^2 \quad \text{and} \quad M_H^2 = 2v^2(\zeta + 2\lambda v^2) \quad . \tag{10}$$

The conditions (5) and (7) imply the following lower-bound (see eq.(10)) for the Higgs mass :

$$M_H^2 \geq \frac{3}{4}\frac{\zeta^2}{\lambda} \quad . \tag{11}$$

Therefore, a *massless* Higgs is out of the model we consider here. A massless Higgs would be present in the spectrum if $\mu^2 > \frac{3}{16}\frac{\zeta^2}{\lambda}$. But, in such a situation, the minima realising the spontaneous symmetry breaking would not be absolute ones, corresponding therefore to metastable ground states, that we avoid here. One-particle states would decay with a short decay-rate if compared to an absolute minimum ground state.

In order to preserve the manifest renormalisability of the model, the 't Hooft gauge [20] is adopted :

$$\hat{S}_{R_\xi}^{\mathrm{gf}} = \int d^3x \left\{ -\frac{1}{2\xi}\Big(\partial^m A_m - \sqrt{2}\xi M_A \theta\Big)^2 \right\} \quad , \tag{12}$$

where ξ is a dimensionless gauge parameter.

By replacing the parametrisation (8) into the action (1), and adding up the 't Hooft gauge (12), it can be directly found the following complete parity-preserving action :

$$\begin{aligned} S_{\mathrm{QED}}^{\mathrm{SSB}} = \int d^3x \Big\{ & -\frac{1}{4}F^{mn}F_{mn} + \frac{1}{2}M_A^2 A^m A_m + \overline{\psi}_+(i\partial\!\!\!/ - m)\psi_+ + \overline{\psi}_-(i\partial\!\!\!/ + m)\psi_- + \\ & + \partial^m H\partial_m H - M_H^2 H^2 + \partial^m\theta\partial_m\theta - M_\theta^2\theta^2 - \frac{1}{2\xi}(\partial^m A_m)^2 + \\ & - qg\overline{\psi}_+ A\!\!\!/ \psi_+ - qg\overline{\psi}_- A\!\!\!/ \psi_- - y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)(2vH + H^2 + \theta^2) + \\ & + Q^2g^2A^mA_m(2vH + H^2 + \theta^2) + 2QgA^m(H\partial_m\theta - \theta\partial_m H) + \\ & - c_3H^3 - c_4H^4 - c_5H^5 - c_6H^6 - c_7\theta^4 - c_8\theta^6 - c_9H\theta^2 - c_{10}H^2\theta^2 + \\ & - c_{11}H^3\theta^2 - c_{12}H^4\theta^2 - c_{13}H\theta^4 - c_{14}H^2\theta^4 \Big\} \quad , \end{aligned} \tag{13}$$

where the constants M_θ^2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13} and c_{14} are defined by

$$\begin{aligned} & M_\theta^2 = \xi M_A^2 \quad , \quad c_3 = 2v(\zeta + \frac{10}{3}\lambda v^2) \quad , \quad c_4 = \frac{\zeta}{2} + 5\lambda v^2 \quad , \quad c_5 = 2\lambda v \quad , \\ & c_6 = \frac{\lambda}{3} \quad , \quad c_7 = \frac{\zeta}{2} + \lambda v^2 \quad , \quad c_8 = \frac{\lambda}{3} \quad , \quad c_9 = 2v(\zeta + 2\lambda v^2) \quad , \\ & c_{10} = \zeta + 6\lambda v^2 \quad , \quad c_{11} = 4\lambda v \quad , \quad c_{12} = \lambda \quad , \quad c_{13} = 2\lambda v \quad \text{and} \quad c_{14} = \lambda \quad . \end{aligned} \tag{14}$$

The Møller scattering to be contemplated will include the scatterings mediated by the gauge field and the Higgs (A_m and H). The scattered electrons exhibit opposite spin polarisations ($e^-_{(\pm)}$ and $e^-_{(\mp)}$) and the same spin polarisations ($e^-_{(\pm)}$ and $e^-_{(\pm)}$). The study of electrons scattered with opposite spin polarisations is motivated by the fact that, in 4-dimensional space-time, a Cooper pair bound state (s-wave state) [8] is built up by a scattering between electrons correlated in phase-space with opposite spins. The interactions involved in such a process are the electromagnetic and the phononic ones. The former is mediated by photons, with a repulsive behaviour, and the latter is mediated by the phonons, which is attractive. The opposite behaviour of these interactions plays a central rôle for the BCS-superconductivity phenomena [6] (weak-coupling superconductors), since, at temperatures below the critical one (T_c), the interaction mediated by phonons (attractive) is stronger than the electromagnetic (repulsive) interaction. For temperatures above T_c, the superconducting phase is destroyed, which means that the net interaction becomes repulsive. On the other hand, the study of the scatterings between electrons with the same polarisation is well-motivated in connection with the phenomenology of superconductors that exhibit high critical magnetic fields as well as the re-entrant superconductivity effect. It is suspected that the resistence of their superconducting phase in presence of very high magnetic fields is caused by p-electron pairing.

For a 3-dimensional space-time, we are now trying to understand, with the help of the model proposed here, what happens if we consider electrons scattered in s and p processes. One of the questions to be answered is whether or not there is a net attractive interaction in $e^-_{(\pm)}-e^-_{(\mp)}$ and $e^-_{(\pm)}-e^-_{(\pm)}$ scatterings, as mediated by the gauge field and the Higgs. Another interesting point to be analised concerns the influence of spin polarisations ($+$ and $-$) on the dynamical nature of these scattering processes.

3 Scattering potentials

To compute the scattering amplitudes, it will be necessary to derive the Feynman rules for propagators and interaction vertices involving the fermions, the gauge field and the Higgs. From the action (13), the following propagator and vertex Feynman rules come out :

1. fermions and Higgs propagators :

$$\langle\overline{\psi}_+\psi_+\rangle = i\frac{\not{k}+m}{k^2-m^2} \quad , \quad \langle\overline{\psi}_-\psi_-\rangle = i\frac{\not{k}-m}{k^2-m^2} \quad \text{and} \quad \langle HH\rangle = \frac{i}{2}\frac{1}{k^2-M_H^2} \quad ; \qquad (15)$$

2. gauge field propagator :

$$\langle A_m A_n\rangle = -i\left[\frac{1}{(k^2-M_A^2)}\left(\eta_{mn}-\frac{k_m k_n}{M_A^2}\right)+\frac{1}{M_A^2}\left(\frac{k_m k_n}{k^2-\xi M_A^2}\right)\right] \quad ; \qquad (16)$$

3. vertex Feynman rules :

$$\mathcal{V}_{+H+} = 2iyv \quad , \quad \mathcal{V}_{-H-} = -2iyv \quad , \quad \mathcal{V}^m_{+A+} = iqg\gamma^m \quad \text{and} \quad \mathcal{V}^m_{-A-} = iqg\gamma^m \quad . \qquad (17)$$

It should be noticed that the convention addopted, $\mathcal{V}_{+H+}$, means the vertex Feynman rule for the interaction term, $\overline{\psi}_+H\psi_+$. This convention is addopted similarly for the other interaction vertices above.

The s-channel amplitudes for the $e^-_{(\pm)}-e^-_{(\mp)}$ and $e^-_{(\pm)}-e^-_{(\pm)}$ scatterings by the gauge field and Higgs, are listed below :

1. scattering amplitude by A_m :

$$-i\mathcal{M}_{\pm A\mp} = \overline{u}_\pm(p_1)\left[iqg\gamma^m_{(\pm)}\right]u_\pm(p_1')\left\{-i\tfrac{\eta_{mn}}{k^2-M_A^2}\right\}\overline{u}_\mp(p_2)\left[iqg\gamma^n_{(\mp)}\right]u_\mp(p_2') \;\; ; \quad (18.a)$$

$$-i\mathcal{M}_{\pm A\pm} = \overline{u}_\pm(p_1)\left[iqg\gamma^m_{(\pm)}\right]u_\pm(p_1')\left\{-i\tfrac{\eta_{mn}}{k^2-M_A^2}\right\}\overline{u}_\pm(p_2)\left[iqg\gamma^n_{(\pm)}\right]u_\pm(p_2') \;\; ; \quad (18.b)$$

2. scattering amplitude by H :

$$-i\mathcal{M}_{\pm H\mp} = \overline{u}_\pm(p_1)\left[\pm 2iyv\right]u_\pm(p_1')\left\{\tfrac{i}{2}\tfrac{1}{k^2-M_H^2}\right\}\overline{u}_\mp(p_2)\left[\mp 2iyv\right]u_\mp(p_2') \;\; ; \quad (19.a)$$

$$-i\mathcal{M}_{\pm H\pm} = \overline{u}_\pm(p_1)\left[\pm 2iyv\right]u_\pm(p_1')\left\{\tfrac{i}{2}\tfrac{1}{k^2-M_H^2}\right\}\overline{u}_\pm(p_2)\left[\pm 2iyv\right]u_\pm(p_2') \;\; , \quad (19.b)$$

where $k^2{=}(p_1-p_1')^2$ is the invariant squared momentum transfer. The Dirac spinors, u_+ and u_-, are the positive-energy solutions to the Dirac equations for ψ_+ and ψ_- (see the Appendix), and they are normalised to :

$$\overline{u}_+(p)u_+(p) = 1 \quad \text{and} \quad \overline{u}_-(p)u_-(p) = -1 \quad . \qquad (20)$$

As discussed in detail in the Appendix, we should stress here that the wave functions u_+ and u_- refer both to the particle (electron) with opposite spins, whereas v_+ and v_- describe both the anti-particle (positron) with opposite spins. In our case, we are actually computing the scattering of 2 electrons with the opposite ($e^-_{(\pm)}$ and $e^-_{(\mp)}$) and the same ($e^-_{(\pm)}$ and $e^-_{(\pm)}$) spins.

To compute the scattering potentials for the interaction between electrons with opposite spin polarisations ($e^-_{(\pm)}$ and $e^-_{(\mp)}$) and with the same spin polarisations ($e^-_{(\pm)}$ and $e^-_{(\pm)}$), we refer to the works of Sucher *et al.* [11], where the concept of potential in quantum field theory and in scattering processes is discussed in great detail.

The calculation of scattering potentials will be performed in the center-of-mass frame, for in this frame the electrons scattered are correlated in momentum space.

By using the Feynman rules displayed above (eqs.(15), (16) and (17)), the following scattering potentials for the $e^-_{(\pm)}-e^-_{(\mp)}$ and $e^-_{(\pm)}-e^-_{(\pm)}$ scattering processes (s and p-wave processes) mediated by the gauge field and the Higgs are found in the center-of-mass frame (*c.m.*): [4]

1. gauge field scattering potential :

$$\mathcal{U}^{c.m.}_{\pm A\mp}(\vec{r}) = q^2g^2\beta_{(\pm)}\beta_{(\mp)}\gamma^m_{(\pm)}\gamma^{(\mp)}_m\int\frac{d^2\vec{q}}{(2\pi)^2}\frac{1}{\vec{q}^2+M_A^2}e^{i\vec{q}.\vec{r}}$$

[4]In the *c.m.* frame, the squared momentum transfer is given by $k^2{=}-\vec{q}^2$. The notations, $\mathcal{U}_{\pm A\mp}(\vec{r})$, $\mathcal{U}_{\pm H\mp}(\vec{r})$, $\mathcal{U}_{\pm A\pm}(\vec{r})$ and $\mathcal{U}_{\pm H\pm}(\vec{r})$, with $r{\equiv}|\vec{r}|$, refer to the scattering potentials (in configuration space) for the processes $e^-_{(\pm)}-e^-_{(\mp)}$ and $e^-_{(\pm)}-e^-_{(\pm)}$, mediated by gauge field and Higgs. The product $\beta_{(\pm)}\beta_{(\mp)}$ is a spinorial factor in the space of the electrons $e^-_{(\pm)}$ and $e^-_{(\mp)}$: $\beta_{(+)}{=}\gamma^0_{(+)}$, $\beta_{(-)}{=}-\gamma^0_{(-)}$ and $\vec{\alpha}_{(\pm)}{\equiv}\gamma^0_{(\pm)}\vec{\gamma}_{(\pm)}$.

$$= -q^2 g^2 \gamma^0_{(\pm)} \gamma^0_{(\mp)} \gamma^m_{(\pm)} \gamma^{(\mp)}_m K_0(M_A r)$$

$$= -q^2 g^2 \left[\mathbb{1} - \vec{\alpha}_{(\pm)} . \vec{\alpha}_{(\mp)} \right] K_0(M_A r) \quad ; \tag{21.a}$$

$$\mathcal{U}^{c.m.}_{\pm A \pm}(\vec{r}) = q^2 g^2 \beta_{(\pm)} \beta_{(\pm)} \gamma^m_{(\pm)} \gamma^{(\pm)}_m \int \frac{d^2\vec{q}}{(2\pi)^2} \frac{1}{\vec{q}^2 + M_A^2} e^{i\vec{q}.\vec{r}}$$

$$= q^2 g^2 \gamma^0_{(\pm)} \gamma^0_{(\pm)} \gamma^m_{(\pm)} \gamma^{(\pm)}_m K_0(M_A r)$$

$$= q^2 g^2 \left[\mathbb{1} - \vec{\alpha}_{(\pm)} . \vec{\alpha}_{(\pm)} \right] K_0(M_A r) \quad ; \tag{21.b}$$

The minus sign in (21.a) deserves some attention. It is due to the fact that $\beta_{(-)} = -\gamma^0_{(-)}$. This is a peculiarity of (1+2)-dimensions: ψ_+ and ψ_- have mass terms with opposite signs (therefore, opposite spins, according to [1, 18]) and so, by looking at the Hamiltonians displayed in the Appendix, one reads off β-terms with opposite signs.

2. Higgs scattering potential :

$$\mathcal{U}^{c.m.}_{\pm H \mp}(\vec{r}) = 2y^2 v^2 \beta_{(\pm)} \beta_{(\mp)} \int \frac{d^2\vec{q}}{(2\pi)^2} \frac{1}{\vec{q}^2 + M_H^2} e^{i\vec{q}.\vec{r}}$$

$$= -2y^2 v^2 \left[\gamma^0_{(\pm)} \gamma^0_{(\mp)} \right] K_0(M_H r) \quad , \tag{22.a}$$

$$\mathcal{U}^{c.m.}_{\pm H \pm}(\vec{r}) = -2y^2 v^2 \beta_{(\pm)} \beta_{(\pm)} \int \frac{d^2\vec{q}}{(2\pi)^2} \frac{1}{\vec{q}^2 + M_H^2} e^{i\vec{q}.\vec{r}}$$

$$= -2y^2 v^2 \left[\gamma^0_{(\pm)} \gamma^0_{(\pm)} \right] K_0(M_H r) \quad , \tag{22.b}$$

where $K_0(Mr)$ is the zeroth-order modified Bessel function of the second kind :

$$\int \frac{d^2\vec{q}}{(2\pi)^2} \frac{1}{\vec{q}^2 + M^2} e^{i\vec{q}.\vec{r}} = \frac{1}{2\pi} K_0(Mr) \quad . \tag{23}$$

This Bessel function presents the following asymptotic behaviour in terms of the Compton wave-length $(\frac{1}{M})$:

$$K_0(Mr) \longrightarrow \begin{cases} -\ln(Mr) \quad , \quad Mr \ll 1 \\ \sqrt{\frac{\pi}{2Mr}} e^{-Mr} \quad , \quad Mr \gg 1 \quad . \end{cases} \tag{24}$$

Now, some conditions on the parameters must be set in order to guarantee a net attractive interaction between scattered electrons with opposite and equal spin polarisations, s and p-wave scattering, respectively. To do that, one assumes the following fine tunning among the parameters :

$$Q^2 g^2 = \zeta + 2\lambda v^2 \quad \text{and} \quad q^2 g^2 < 2y^2 v^2 \quad . \tag{25}$$

From the conditions above, and the conditions given by eqs.(5), (6), (10) and (11), after some algebraic manipulations, an interesting inequality arises :

$$\frac{Q^2}{q^2} > \frac{\lambda}{3y^2} \quad ; \tag{26}$$

where it does not depend only on the fundamental constant, g (the electromagnetic coupling constant), but on the matter self-couplings.

The coherence length of a Cooper pair, as Cooper found out for the 2-electron bound state [8], is much bigger than the electron Compton wave-length ($mr \gg 1$), namely, the former is of order 10^4Å and the latter of 10^{-2}Å. Therefore, for the sake of studying the possible existence of a possible existence of s and p electron-pair condensates in the parity-preserving QED$_3$ discussed throughout this work, the strength of the net scattering potentials in s and p scattering processes between electrons candidates to built up s and p Cooper pairs, read

$$\mathcal{V}^{s}_{c.m.}(r) = -\left[2y^2v^2 + q^2g^2\right]\sqrt{\frac{\pi}{2M_H r}}e^{-M_H r} \quad ; \tag{27}$$

$$\mathcal{V}^{p}_{c.m.}(r) = -\left[2y^2v^2 - q^2g^2\right]\sqrt{\frac{\pi}{2M_H r}}e^{-M_H r} \quad , \tag{28}$$

where the asymptotic approximation, $M_H r \gg 1$, is compatible with the dimensions through which Cooper pair exists.

Therefore, this result shows that the net attractive $e^-_{(\pm)}$–$e^-_{(\mp)}$ and $e^-_{(\pm)}$–$e^-_{(\pm)}$- scattering potentials (27) and (28) are non-confining, contrary to what happens for massive electrons scattered by massless gauge field, where the potential is completely confining [15]. Therefore, the interactions mediated by the gauge field and the Higgs are attractive in a scattering between electrons with opposite spin polarisations ($e^-_{(\pm)}$–$e^-_{(\mp)}$-scattering). For scatterings with a scalar exchange, the spin polarisations do not affect the behaviour of potential: it will be always attractive. This result is expected, since the Higgs particle does not *feel* the electron polarisations.

An interesting point to remark is that, in spite the scattered particles have the same electric charge, the spin polarisation is determinant for the behaviour of the scattering potential for processes where a gauge field is exchanged. In the case where the scattered electrons have opposite spin polarisations (the opposite mass term in Dirac's equation) , the interaction is attractive. However, one should notice that this result is not conflicting with QED expectations. In our model, the photon-mediated interaction takes place on a non-trivial background, set by the Higgs field. Electron interaction is repulsive if the exchanged photon propagates on a QED vacuum. In the model proposed here, the electron mass and the electron interactions are to be referred not to a trivial vacuum, but to a background responsible for the photon mass. Therefore, we interpret the attraction as a byproduct of the physics of electrons propagating on a non-trivial Higgs background. Nevertheless, for scatterings between electrons with the same spin state (the same mass term in Dirac's equation), the interaction becomes repulsive.

4 Discussions and general conclusions

In this work we concentrate efforts in trying to understand many intriguing features of dynamical processes in D=1+2, where a parity-preserving QED$_3$ is coupled to scalar matter. In this scenario, the spontaneous symmetry breaking mechanism of a $U(1)$-symmetry takes place. The breakingdown is realised by a sixth-power potential with a mass generation for the gauge boson and the fermions. The spontaneous symmetry

breaking mechanism is the responsible for the appearance of a kind of Meissner effect, since the scalar magnetic field obeys a London equation: $B(r){=}B_0e^{-\lambda r}$, where $\lambda{=}\frac{1}{M_A}$ is the penetration length. Therefore, as M_A depends on the vacuum expectation value, v, it can be concluded that $\lambda{\to}\infty$ when $v{\to}0$. This means that the Meissner effect is completely destroyed when the scalar assumes a vanishing vacuum expectation value, as in the symmetric regime.

Now, bering in mind the Coleman-Mermin-Wagner theorem [21], an interesting question naturally comes out: is there a critical temperature, T_c, such that for temperatures above the critical one ($T > T_c$) the gauge symmetry is restored? If yes, it follows that the scalar assumes a vanishing vacuum expectation value which leaves the gauge field massless and, as a consequence, the Meissner effect disappears. Therefore, a superconducting-type phase transition should be present as a direct consequence of symmetry restoration by the finite temperature quantum corrections.

An interesting point to be emphasised is the influence of spin polarisations on the dynamical nature of the scattering processes. This feature is dictated by the Poincaré group structure of $D{=}1{+}2$. As a peculiarity of this space-time and the Higgs background on which the electrons and photons propagate, electrons scattered by a massive gauge boson and by a Higgs can experience an attractive interaction. The interaction potential associated to gauge boson exchange displays opposite behaviours when the electrons scattered have opposite or the same spin polarisations, since electrons propagate on a non-trivial Higgs background. For the case where the Higgs is exchanged, the scattering potential is completely insensitive to the electrons polarisation, as is expected, since the Higgs is a spinless particle. One concludes in this work that electrons can attract each other in $D{=}1{+}2$ through scattering processes where a massive gauge boson and a Higgs are involved. This attraction between electrons might favour a bound state. As long as the behaviour of this model at the quantum level is concerned, it shows to be stable under radiative corrections and anomaly free in the symmetric and broken regimes, which proves its renormalisability [16].

It should be pointed out that, in order to be sure of the existence of a bound state in such scatterings, it is more advisable to study the Bethe-Salpeter [12] equation in $D{=}1{+}2$ [13] for the model proposed here. Such an analysis is more reliable in view of its intrinsically non-perturbative nature. It is worthwhile to stress that our results simply suggest that, at the semiclassical level, a net attractive interaction between electrons with opposite polarisations might point out pair condensation if Bethe-Salpeter equations are taken into account [13]. On the other hand, if an attraction is felt at the level of tree amplitudes, we would not expect that loop corrections, that bring about powers of $\hbar$, might work against pair condensation. In any case, to our mind, it would be more reasonable to pursue an investigation of the Bethe-Salpeter equations (rather than computing higher-loop corrections) in order to infer about electron-pair condensation in the model discussed throughout this paper [14].

As a final remark, we point out that the finite temperature approach could be of interest in order to verify whether or not there are pair-condensation phase transitions for some critical temperature, T_c, in the cases of s and p bound states. If no more bound states exist in the solutions of the Bethe-Salpeter equation for temperatures above the critical one ($T > T_c$), the electrons are no more correlated and, therefore, the gauge symmetry is restored; as a consequence, the Meissner effect disappears.

A Some properties of Dirac spinors in D=3

In this Appendix, we present some aspects of Dirac spinors living in D=3, like the positive and negative energy solutions to the Dirac equations satisfied by ψ_+ and ψ_-. We state clearly the connection between mass and spin and, in order to elucidate some peculiarities of electrons scattering in 3 space-time dimensions, we present the Hamiltonian for both ψ_+ and ψ_-. We also compute explicitly the charges of the positive and negative energy wave functions associated to ψ_+ and ψ_-

A.1 Positive and negative energy solutions for ψ_+ and ψ_-

Let us consider u_+ and v_+, u_- and v_-, respectively, as the positive and negative solutions to the Dirac equations for ψ_+ and ψ_-. Therefore, they satisfy the following equations in momentum space :

$$(\not{p} - m)u_+(p) = 0 \ , \quad (-\not{p} - m)v_+(p) = 0 \ ; \tag{A.1}$$

$$(\not{p} + m)u_-(p) = 0 \ , \quad (-\not{p} + m)v_-(p) = 0 \ . \tag{A.2}$$

Their solutions are given by

$$u_+(p) = \frac{\not{p} + m}{\sqrt{2m(m+E)}}\, u_+(m,\vec{0}) \ , \quad v_+(p) = \frac{-\not{p} + m}{\sqrt{2m(m+E)}}\, v_+(m,\vec{0}) \ ; \tag{A.3}$$

$$u_-(p) = \frac{-\not{p} + m}{\sqrt{2m(m+E)}}\, u_-(m,\vec{0}) \ , \quad v_-(p) = \frac{\not{p} + m}{\sqrt{2m(m+E)}}\, v_-(m,\vec{0}) \ , \tag{A.4}$$

where $E \equiv k^0 = \sqrt{\vec{k}^2 + m^2} > 0$. The wave functions $u_+(m,\vec{0})$, $v_+(m,\vec{0})$, $u_-(m,\vec{0})$ and $v_-(m,\vec{0})$ are the solutions of eqs.(A.1-A.2) in the rest frame

$$u_+(m,\vec{0}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \ , \quad v_+(m,\vec{0}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \ ; \tag{A.5}$$

$$u_-(m,\vec{0}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \ , \quad v_-(m,\vec{0}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \ . \tag{A.6}$$

The positive and negative energy solutions given by eqs.(A.3-A.4) are normalised to :

$$\overline{u}_+(p)u_+(p) = 1 \ , \quad \overline{v}_+(p)v_+(p) = -1 \ ; \tag{A.7}$$

$$\overline{u}_-(p)u_-(p) = -1 \ , \quad \overline{v}_-(p)v_-(p) = 1 \ . \tag{A.8}$$

A.2 The spin of u_+, v_+, u_- and v_-

Now, by considering the results of last subsection, one is able to determine the spins of the solutions u_+, v_+, u_- and v_-. We compute the spins in the particle rest frame, since we have in mind to explicitly exhibit the fact that the sign of the mass term fixes the polarisation of the fermion.

In D=3, the generators of the $\overline{SO(1,2)}$ group in the spinor representation read :

$$\Sigma^{kl} = \tfrac{1}{4}\,[\gamma^k, \gamma^l] \ , \tag{A.9}$$

where the γ-matrices are taken as $\gamma^m{=}(\sigma_x, i\sigma_y, -i\sigma_z)$.

The spin operator S^{12} is obtained from (A.9), and it reads

$$S^{12} = \frac{1}{2}\,\sigma_x \quad . \tag{A.10}$$

Its action upon the rest frame wave functions given by eqs.(A.5-A.6) is collected below :

$$S^{12}\ u_+(m,\vec{0}) = s^u_+\ u_+(m,\vec{0}) \quad , \quad S^{12}\ v_+(m,\vec{0}) = s^v_+\ v_+(m,\vec{0}) \quad ; \tag{A.11}$$
$$S^{12}\ u_-(m,\vec{0}) = s^u_-\ u_-(m,\vec{0}) \quad , \quad S^{12}\ v_-(m,\vec{0}) = s^v_-\ v_-(m,\vec{0}) \quad . \tag{A.12}$$

With the help of (A.5-A.6) and (A.10), we find the following values for the spin eigenvalues s^u_+, s^u_-, s^v_+, and s^v_- :

$$s^u_+ = \frac{1}{2} \quad , \quad s^u_- = -\frac{1}{2} \quad , \quad s^v_+ = -\frac{1}{2} \quad , \quad s^v_- = \frac{1}{2} \quad . \tag{A.13}$$

From eq.(A.13), it can be concluded that electrons (u_+ and u_-) and positrons (v_+ and v_-) with opposite mass terms have opposite spin polarisations. It should be pointed out that this result is in completely agreement with ref.[18]. In the Section A.5 of this Appendix we prove explicitly that the wave functions u_+ and u_- are associated to electrons whereas v_+ and v_- are associated to positrons.

An interesting point to stress here concerns the polarisations of a particle (u) and the corresponding anti-particle (v) belonging to the same Dirac spinor (ψ). As a typical feature of 3 space-time dimensions, if a particle has spin s, its anti-particle has spin $-s$.

A.3 The Hamiltonian for ψ_+ and ψ_-

In this subsection, the relation between the opposite mass term signs and the opposite signs of β-matrices respected to ψ_+ and ψ_- becomes clear by computing the free Hamiltonian operator H_0.

For a general massive Dirac spinor, χ, the free Hamiltonian operator in momentum space, H_0, is given by :

$$H_0\chi \equiv (\vec{\alpha}.\vec{p} + \beta m)\chi \quad , \tag{A.14}$$

where

$$\vec{\alpha} = \gamma^0\vec{\gamma} \quad \text{and} \quad \beta = \gamma^0 \quad . \tag{A.15}$$

Now, considering the Dirac equations for ψ_+ and ψ_- :

$$(i\partial\!\!\!/ - m)\psi_+ = 0 \quad \text{and} \quad (i\partial\!\!\!/ + m)\psi_- = 0 \quad , \tag{A.16}$$

it follows that

$$i\frac{\partial}{\partial t}\psi_+ = \left(i\gamma^0_{(+)}\vec{\gamma}_{(+)}.\vec{\partial} + \beta_{(+)}m\right)\psi_+ \equiv H_0^{(+)}\psi_+ \quad ; \tag{A.17}$$
$$i\frac{\partial}{\partial t}\psi_- = \left(i\gamma^0_{(-)}\vec{\gamma}_{(-)}.\vec{\partial} + \beta_{(-)}m\right)\psi_- \equiv H_0^{(-)}\psi_- \quad . \tag{A.18}$$

Therefore, in momentum space, the Hamiltonians $H_0^{(+)}$ and $H_0^{(-)}$ read

$$H_0^{(+)}\psi_+ = \left(\vec{\alpha}_{(+)}.\vec{p} + \beta_{(+)}m\right)\psi_+ \quad ; \tag{A.19}$$
$$H_0^{(-)}\psi_- = \left(\vec{\alpha}_{(-)}.\vec{p} + \beta_{(-)}m\right)\psi_- \quad , \tag{A.20}$$

where, from (A.14), it can be concluded that

$$\vec{\alpha}_{(+)} = \gamma^0_{(+)}\vec{\gamma}_{(+)} \quad \text{and} \quad \beta_{(+)} = \gamma^0_{(+)} \quad ; \tag{A.21}$$

$$\vec{\alpha}_{(-)} = \gamma^0_{(-)}\vec{\gamma}_{(-)} \quad \text{and} \quad \beta_{(-)} = -\gamma^0_{(-)} \quad . \tag{A.22}$$

The eqs.(A.21-A.22) completely determine the scattering potentials behaviour for the scattering processes of $e^-_{(\pm)}-e^-_{(\mp)}$ and $e^-_{(\pm)}-e^-_{(\pm)}$ mediated by the gauge field and the Higgs. They are in agreement to the fact that the Higgs scattering potential does not *feel* the electron polarisations, since Higgs is spinless. It is only possible if eqs.(A.21-A.22) are fulfiled.

A.4 The spin of u_+, v_+, u_- and v_- as a quantum number

Let us consider the spin operator given by eq.(A.10) :

$$S^{12} = \frac{1}{2}\,\sigma_x \quad ,$$

and the free Hamiltonian operators in momentum space for the spinors ψ_+ and ψ_- (eqs.(A.19-A.20)) :

$$H_0^{(+)} = \left(\vec{\alpha}_{(+)}.\vec{p} + \beta_{(+)}m\right) \quad ,$$

$$H_0^{(-)} = \left(\vec{\alpha}_{(-)}.\vec{p} + \beta_{(-)}m\right) \quad ,$$

where $\vec{\alpha}_{(\pm)}$ and $\beta_{(\pm)}$ are given by eqs.(A.21-A.22). It can be easily shown that the following commutators vanish

$$\left[H_0^{(+)}, S^{12}\right] = 0 \quad , \tag{A.23}$$

$$\left[H_0^{(-)}, S^{12}\right] = 0 \quad . \tag{A.24}$$

This result ensures that the eigenvalues (s^u_+, s^v_+, s^u_- and s^v_-) of the spin operator, S^{12}, corresponding respectively to the wave functions u_+, v_+, u_- and v_- are indeed good quantum numbers to label physical states.

A.5 The charges of u_+, v_+, u_- and v_-

In order to determine the charges of the particles associated to the wave functions, u_+, v_+, u_- and v_-, it is necessary to compute the eigenvalues of the charge operators, Q_+ and Q_-, respected to the field operators, ψ_+ and ψ_-. Their expansion in terms of the creation and annihilation operators read as below :

$$\psi_+(x) = \int \frac{d^2\vec{k}}{(2\pi)^2}\frac{m}{k^0}\left[a_+(k)u_+(k)e^{-ik.x} + b^\dagger_+(k)v_+(k)e^{ik.x}\right] \quad , \tag{A.25}$$

$$\psi_-(x) = \int \frac{d^2\vec{k}}{(2\pi)^2}\frac{m}{k^0}\left[a_-(k)u_-(k)e^{-ik.x} + b^\dagger_-(k)v_-(k)e^{ik.x}\right] \quad , \tag{A.26}$$

$$\overline{\psi}_+(x) = \int \frac{d^2\vec{k}}{(2\pi)^2}\frac{m}{k^0}\left[a^\dagger_+(k)\overline{u}_+(k)e^{ik.x} + b_+(k)\overline{v}_+(k)e^{-ik.x}\right] \quad , \tag{A.27}$$

$$\overline{\psi}_-(x) = \int \frac{d^2\vec{k}}{(2\pi)^2}\frac{m}{k^0}\left[a^\dagger_-(k)\overline{u}_-(k)e^{ik.x} + b_-(k)\overline{v}_-(k)e^{-ik.x}\right] \quad , \tag{A.28}$$

where the operators, $a_+^\dagger$, $b_+^\dagger$, $a_-^\dagger$ and $b_-^\dagger$, are the creation operators, and, a_+, b_+, a_- and b_-, are the annihilation operators. The wave functions were analysed in details in the Section A.1 of this Appendix.

With the help of the Dirac equations (A.1- A.2), the normalisation conditions (A.7-A.8) and the relation

$$\left\{ \not{p}, \gamma^0 \right\} = 2p^0 \quad , \tag{A.29}$$

the following equations are satisfied by the wave functions u_+, v_+, u_- and v_- :

$$u_+^\dagger(p)u_+(p) = \frac{p^0}{m} \quad , \quad v_+^\dagger(p)v_+(p) = \frac{p^0}{m} \quad ; \tag{A.30}$$

$$u_-^\dagger(p)u_-(p) = \frac{p^0}{m} \quad , \quad v_-^\dagger(p)v_-(p) = \frac{p^0}{m} \quad . \tag{A.31}$$

The microcausality fixes the following anticommutation relations :

$$\left\{ \psi_+(x), \psi_+^\dagger(y) \right\}_{x^0=y^0} = \delta^2(\vec{x}-\vec{y}) \quad , \quad \left\{ \psi_-(x), \psi_-^\dagger(y) \right\}_{x^0=y^0} = \delta^2(\vec{x}-\vec{y}) \quad . \tag{A.32}$$

Now, by assuming the field operator expansions (A.25-A.28), and the normalisation conditions given by eqs.(A.30-A.31), the anticommutation relations between the creation and annihilation operators read :

$$\left\{ a_+(k), a_+^\dagger(p) \right\} = (2\pi)^2 \, \frac{k^0}{m} \, \delta^2(\vec{k}-\vec{p}) \quad , \tag{A.33}$$

$$\left\{ b_+(k), b_+^\dagger(p) \right\} = (2\pi)^2 \, \frac{k^0}{m} \, \delta^2(\vec{k}-\vec{p}) \quad , \tag{A.34}$$

$$\left\{ a_-(k), a_-^\dagger(p) \right\} = (2\pi)^2 \, \frac{k^0}{m} \, \delta^2(\vec{k}-\vec{p}) \quad , \tag{A.35}$$

$$\left\{ b_-(k), b_-^\dagger(p) \right\} = (2\pi)^2 \, \frac{k^0}{m} \, \delta^2(\vec{k}-\vec{p}) \quad . \tag{A.36}$$

The charge operators, Q_+ and Q_-, associated to the field operators, ψ_+ and ψ_-, are defined by the following normal ordering products :

$$Q_+ = \int d^2\vec{x} : j_+^o(x) := -qg \int d^2\vec{x} : \psi_+^\dagger(x)\psi_+(x) : \quad , \tag{A.37}$$

$$Q_- = \int d^2\vec{x} : j_-^o(x) := -qg \int d^2\vec{x} : \psi_-^\dagger(x)\psi_-(x) : \quad , \tag{A.38}$$

which in terms of the creation and annihilation operators are given by

$$Q_+ = -qg \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{m}{k^0} \left[a_+^\dagger(k)a_+(k) - b_+^\dagger(k)b_+(k) \right] \quad , \tag{A.39}$$

$$Q_- = -qg \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{m}{k^0} \left[a_-^\dagger(k)a_-(k) - b_-^\dagger(k)b_-(k) \right] \quad . \tag{A.40}$$

From the anticommutation relations (A.33-A.36) and the eqs.(A.39-A.40), for the charge operators Q_+ and Q_-, it can be easily shown that

$$\left[Q_+, a_+^\dagger(p) \right] = -qg \; a_+^\dagger(p) \quad , \quad \left[Q_+, b_+^\dagger(p) \right] = +qg \; b_+^\dagger(p) \quad , \tag{A.41}$$

$$\left[Q_-, a_-^\dagger(p) \right] = -qg \; a_-^\dagger(p) \quad , \quad \left[Q_-, b_-^\dagger(p) \right] = +qg \; b_-^\dagger(p) \quad . \tag{A.42}$$

Let us denote the vacuum ground state by the "ket", $|0\rangle$, such that

$$a_+(k)|0\rangle = 0 \;\;, \quad b_+(k)|0\rangle = 0 \quad , \tag{A.43}$$

$$a_-(k)|0\rangle = 0 \;\;, \quad b_-(k)|0\rangle = 0 \quad , \tag{A.44}$$

where $\langle 0|0\rangle = 1$. Now, bearing in mind the commutation relations given by eqs.(A.41-A.42), and applying them to the vacuum state, it follows that

$$Q_+|e^-_{(+)}\rangle = -qg\ |e^-_{(+)}\rangle \quad \text{where} \quad |e^-_{(+)}\rangle = a^\dagger_+|0\rangle \quad ; \tag{A.45}$$

$$Q_+|e^+_{(+)}\rangle = +qg\ |e^+_{(+)}\rangle \quad \text{where} \quad |e^+_{(+)}\rangle = b^\dagger_+|0\rangle \quad ; \tag{A.46}$$

$$Q_-|e^-_{(-)}\rangle = -qg\ |e^-_{(-)}\rangle \quad \text{where} \quad |e^-_{(-)}\rangle = a^\dagger_-|0\rangle \quad ; \tag{A.47}$$

$$Q_-|e^+_{(-)}\rangle = +qg\ |e^+_{(-)}\rangle \quad \text{where} \quad |e^+_{(-)}\rangle = b^\dagger_-|0\rangle \quad . \tag{A.48}$$

Due to these results, one concludes that :

1. $a^\dagger_+$ creates an electron (u_+) with spin $s^u_+ = \frac{1}{2}$ and charge $-qg$.

2. $b^\dagger_+$ creates a positron (v_+) with spin $s^v_+ = -\frac{1}{2}$ and charge $+qg$.

3. $a^\dagger_-$ creates an electron (u_-) with spin $s^u_- = -\frac{1}{2}$ and charge $-qg$.

4. $b^\dagger_-$ creates a positron (v_-) with spin $s^v_- = \frac{1}{2}$ and charge $+qg$.

As a final conclusion, u_+ and u_- are wave functions of electrons with opposite spins ($e^-_{(+)}$ and $e^-_{(-)}$), whereas v_+ and v_- are wave functions of positrons with opposite spins ($e^+_{(+)}$ and $e^+_{(-)}$), which is in completely agreement with the fact that spin is related to a space-time symmetry (Lorentz group) and electric charge is related to an internal symmetry (gauge symmetry). Some of the physical relevant results obtained in this Appendix are summarised in Table 1.

Creation operator	Charge operator	Charge	Particle	Symbol	Wave function	Spin
$a^\dagger_+$	Q_+	$-qg$	electron	$e^-_{(+)}$	u_+	$s^u_+ = +\frac{1}{2}$
$a^\dagger_-$	Q_-	$-qg$	electron	$e^-_{(-)}$	u_-	$s^u_- = -\frac{1}{2}$
$b^\dagger_+$	Q_+	$+qg$	positron	$e^+_{(+)}$	v_+	$s^v_+ = -\frac{1}{2}$
$b^\dagger_-$	Q_-	$+qg$	positron	$e^+_{(-)}$	v_-	$s^v_- = +\frac{1}{2}$

Table 1: Charge and spin of the particles associated to the field operators, ψ_+ and ψ_-.

All results presented in this Appendix show non-trivial aspects of parity-preserving QED_3. The relation between the signal of spin and the signal of mass in the Dirac mass term is an interesting feature of massive fermions in D=1+2. Another point is the fact that, due to Higgs mechanism, the interaction potential experienced by the electrons of both possible polarisations are non-confining, contrary to the case where massless gauge field are taken into account [15]. It provides a net attractive interaction between electrons of both spins, which might favour an electron-pair condensation of s and p-wave type.

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